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Confinement Properties of Non-neutral and Neutral Plasmas in an Axially Symmetric System

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Abstract. A general approach is established to examine confinement properties of both non-neutral and neutral axially symmetric plasmas. In this approach, problems are reduced to finding total minimum energy states under the conservation of total angular momentum and total number of particles. First, the well-known result of a non-neutral plasma in a cylindrical system is reproduced for checking the approach. Second, the approach is applied to analysis of confinement properties of tokamak plasmas with a transport barrier and it is shown that the tokamak with an internal transport barrier is in the minimum energy state.

INTRODUCTION

It is well known that a single species plasma in a Penning trap has a very long lifetime. This long confinement can be understood by the constancy of the total canonical angular momentum in a cylindrical system [1, 2]. The total canonical angular momentum is given by $P_\theta = \sum_j (m_j u_{\theta j} r_j + e_j A_{\theta j} r_j)$, where the first term is the kinetic term of the angular momentum and the second term is the vector potential term of the angular momentum. In the case of a single species plasma, the vector potential term is much larger than the kinetic term. Then, the total angular momentum in a uniform magnetic field can be written as $P_\theta = \frac{eB}{2} \sum_j r_j^2$. Accordingly, the location of the single species plasma particles is constrained, and this is considered as the essence of the long confinement.

On the other hand, in a case of a neutral plasma, pairs of ions and electrons can escape together from the system without changing the vector potential term of the total angular momentum. This is considered to be the reason why the confinement of neutral plasmas is poor. However, the situation changes when the mechanical part of the total angular momentum dominates over the vector potential term. Then, the constancy of the total

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canonical angular momentum places a constraint similar to that for the single species plasma.

In this paper, we establish an approach to examine confinement properties of both non-neutral and neutral plasmas under the constraint of conservation of the total angular momentum. First, the approach is applied to a cylindrical single species plasma in order to check its validity. Then, the approach is applied to a tokamak plasma with a transport barrier.

CYLINDRICAL SINGLE SPECIES PLASMA

In the case of a cylindrical single species plasma, the total angular momentum P_θ and the total number of particles N per unit length may be given by

$$P_\theta = 2\pi\int_0^a n(mu_\theta + eA_\theta)r^2 dr, \quad N = 2\pi\int_0^a nrdr. \tag{1}$$

The total energy U per unit length can also be conserved, where U is the sum of the total kinetic energy K and the total potential energy W, i.e., $U = K + W$. The total kinetic energy and the total potential energy can be written as

$$K = \pi\int_0^a nmu_\theta{}^2 rdr, \quad W = 2\pi\int_0^a (\frac{3}{2}nkT + \frac{1}{2}\varepsilon_0 E^2 + \frac{B^2}{2\mu_0})rdr. \tag{2}$$

The plasma equilibrium under the conservation of the total angular momentum and the total number of particles can be obtained by finding the state for the minimum total potential energy W. This problem is equivalent to finding the state for the maximum kinetic energy K when the total energy U is conserved.

The force balance equation is given by

$$E + u_\theta B = -\frac{1}{en}\frac{d(nkT)}{dr}, \tag{3}$$

where E is the radial component of the electric field. Eliminating u_θ from Eq. (2) and Eq. (3), one rewrites the total kinetic energy as

$$K = \pi \int_0^a nm(E + \frac{1}{en}\frac{d(nkT)}{dr})^2 r dr. \tag{4}$$

Since the vector potential term of the total angular momentum for a single species plasma is much larger than the kinetic term, the total angular momentum in a uniform magnetic field is given by

$$P_\theta = \pi \int_0^a neBr^2 dr. \tag{5}$$

By use of the Poison equation

$$n = \frac{\varepsilon_0}{e}\frac{1}{r}\frac{d(rE)}{dr}, \tag{6}$$

Equation (5) becomes

$$P_\theta = \pi\varepsilon_0 B \int_0^a r^2 \frac{d(rE)}{dr} dr. \tag{7}$$

Performing a partial integral of Eq. (7), one obtains

$$P_\theta = \pi\varepsilon_0 B\{E(a)a^2 - 2\int_0^a Er^2 dr\}. \tag{8}$$

The total kinetic energy Eq. (4) can also be rewritten by use of Eq. (6) as

$$K = \frac{\pi\varepsilon_0 m}{eB^2}\int_0^a \frac{d(rE)}{dr}(E + \frac{1}{en}\frac{d(nkT)}{dr})^2 dr. \tag{9}$$

Performing a partial integral, one gets

$$K = \frac{\pi\varepsilon_0 m}{eB^2}\left\{\left[Ea(E + \frac{1}{en}\frac{d(nkT)}{dr})\right]_0^a - \int_0^a (rE)\frac{d}{dr}(E + \frac{1}{en}\frac{d(nkT)}{dr})^2 dr\right\}. \tag{10}$$

Accordingly, the problem is reduced to finding the maximum value of Eq. (10) under the constraint of conservation of the total angular momentum given by Eq. (8). This variation problem is solved by using the Lagrangian undetermined multiplier method:

$$\delta(K - \lambda P_\theta) = 0, \tag{11}$$

where λ is an adjustable constant. Equation (11) leads to

$$\frac{d}{dr}\{(rE)\frac{d}{dr}(E + \frac{1}{en}\frac{d(nkT)}{dr})^2 - \lambda r^2 E\} = 0. \tag{12}$$

Integrating Eq. (12) and using the condition $u_\theta(0) = 0$, one obtains the following relation

$$E + \frac{1}{en}\frac{d(nkT)}{dr} = \pm\sqrt{\frac{\lambda}{2}}r. \tag{13}$$

Substituting Eq. (13) into Eq. (6), one gets

$$\frac{e}{\varepsilon_0}n = \sqrt{2\lambda} - \frac{d}{dr}\frac{r}{en}\frac{d(nkT)}{dr}. \tag{14}$$

A good approximate solution for the plasma density is given by $n = n(0)(1 - e^{(r-a)/\lambda_D})$, where the Debye length $\lambda_D = \sqrt{\varepsilon_0 kT(0)/e^2 n(0)}$. This represents the well-known equilibrium with a density profile nearly constant up to the plasma radius a and sharply falling down to null within a few Debye length [1, 2].

TOROIDALLY SYMMETRIC TOKAMAK PLASMA

In the case of a neutral plasma in a toroidally symmetric system such as a tokamak, the total toroidal angular momentum P_ϕ is conserved. For flux surfaces having a concentric circular cross-section, one may write

$$P_\phi = 4\pi^2 R_0{}^2 \int_0^a nmu_\phi r dr, \quad N = 4\pi^2 R_0 \int_0^a nrdr. \tag{15}$$

where R_0 is the major radius of the axis and N is the total number of particles. The total energy U, the sum of the total kinetic energy K and the total potential energy W, may

also be conserved: $U = K + W$. The total kinetic energy averaged over flux surfaces may be written as $K = 4\pi^2 R_0 \int_0^a nmu^2 r dr$, where $u^2 = u_\phi^2 + u_\theta^2$. The total potential energy averaged over flux surfaces may be written as

$$W = 4\pi^2 R_0 \int_0^a \{\frac{3}{2} nk(T_i + T_e) + \frac{1}{2}\varepsilon_0 E^2 + \frac{B^2}{2\mu_0}\} r dr. \tag{16}$$

Thus, the maximum total kinetic energy corresponds to the minimum total potential energy. Therefore, the problem of finding the minimum potential energy state is reduced to find the maximum kinetic energy state under the constraint of the conserved total density and total angular momentum. This solution is again obtained by the Lagrangian undetermined multiplier method:

$$\delta(K - \lambda_1 N - \lambda_2 P_\phi) = 0, \tag{17}$$

where λ_1 and λ_2 are constant. Equation (17) can be written in the form

$$(u^2 - \lambda_1 - \lambda_2 u_\phi)\frac{\partial n}{\partial u_\phi} + (2u_\phi - \lambda_2)n = 0. \tag{18}$$

The solution for Eq. (18) is given by

$$n = \frac{c}{u^2 - \lambda_1 - \lambda_2 u_\phi}. \tag{19}$$

COMPARISON OF TEORETICAL RESULTS WITH DIII-D DATA

The theoretical result is tested against actual tokamak experimental data. DIII-D plasmas with an internal transport barrier (ITB) are chosen as a test base. Density profiles of the DIII-D plasmas are computed from the measured toroidal angular (rotation) velocities based on Eq. (19). Figure 1 shows the measured density profile of QDB (Quiescent Double Barrier) mode DIII-D plasma as the solid curve [3] together with two examples of calculated theoretical density profiles (dashed and dotted curves). Either of them are a very good fit except for some mismatches near the edge.

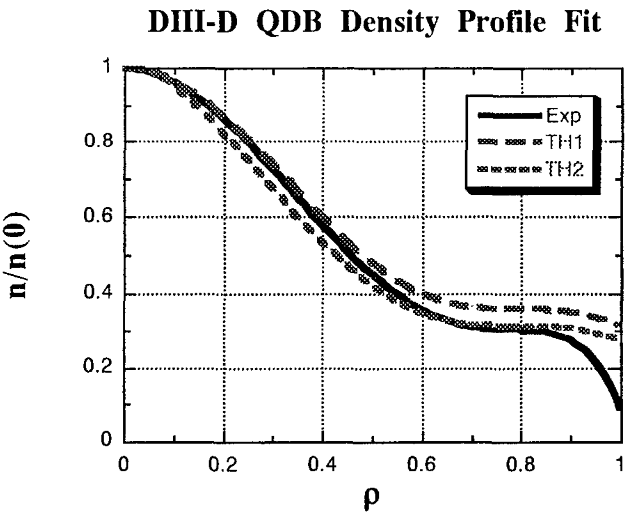

FIGURE 1. The normalized density profile of DIII-D QDB (shot number 106919 at 3 s) shown by the solid curve and two theoretical fits shown by dashed and dotted curves.

This indicates that the DIII-D plasma with an internal transport barrier is indeed a minimum (potential) energy state under the constraint of the conservation of total angular momentum.

CONCLUSION

An approach is established to examine equilibrium profiles for both non-neutral and neutral plasmas under the constraint of conservation of the total angular momentum. One important result obtained from this examination is that a tokamak plasma with an internal transport barrier is in a minimum energy state under the constraint of the conservation of total angular momentum.

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